MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2022/2023 (TERM ID 2215)

JPCM0235 - CALCULUS

18 FEBRUARY 2023
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This Question paper consists of 2 pages excluding the cover page and appendix.
2. Answer **all FIVE** questions. Each question carries equal marks and the distribution of the marks is given.
3. Write all your answers in the Answer Booklet provided.
4. Key formulas are given in Appendix.

Question 1 [10 marks]

a. Find the following limits.

i. $\lim_{x\to 4}\dfrac{x^2+x-20}{x-4}$ (2 marks)

ii. $\lim_{x\to 3}\dfrac{\sqrt{x}-\sqrt{3}}{x^2-3x}$ (2 marks)

iii. $\lim_{x\to -\infty}\dfrac{-x^4}{x^4+1}$ (2 marks)

b. Let $f(x)=\begin{cases} A^2x^2, x\le 2 \\ (1-A)x, x>2 \end{cases}$. For what value of A is $f(x)$ continuous at $x=2$?

(4 marks)

Question 2 [10 marks]

a. Differentiate the following function.

i. $y=\ln\left(x^2+3\right)^2$ (2 marks)

ii. $y^2-x^3=9x$ (2 marks)

b. Evaluate the following integral.

i. $\int\left(e^x+e^{-x}\right)^2dx$ (2 marks)

ii. $\int 3x\sqrt{x^2+4}dx$ (Hint: Use substitution method) (4 marks)

Question 3 [10 marks]

a. Given $f(x)=2+3x-x^3$.

i. Find the intervals of increase and decrease. (3 marks)

ii. Find the intervals of concavity. (3 marks)

iii. Find the x-coordinates of the local maximum and minimum points. (1 mark)

Continued...

b. A thin sheet of ice is in the form of a circle. If the ice is melting in such a way that the area of the sheet is decreasing at a rate of 0.5 m^2/sec at what rate is the radius decreasing when the area of the sheet is 12 m^2?

(3 marks)

Question 4 [10 marks]

a. Find the area of the region in the first quadrant bounded by the curve $x = y^2 - y - 6$ and the line $x = 2y + 4$.

(5 marks)

b. The region bounded by the two curves $y = 2x^2$ and $y = x^3$ is rotated completely about the x-axis. Find the volume of the solid formed.

(5 marks)

Question 5 [10 marks]

a. Find the general solution for the differential equation $y' = \frac{x}{y^2}$.

(3 marks)

b. Find the unique solution for the second order differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 5y = 0 \quad \text{where } y(0) = 1 \text{ and } y'(0) = 0$$

(7 marks)

End of Paper

APPENDIX

A. Differentiation Rules

$\frac{d}{dx}[x^n] = nx^{n-1}$; n is any real number

$\frac{d}{dx}[f(x).g(x)] = f(x)g'(x) + f'(x)g(x)$; The Product Rule

$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$; The Quotient Rule

$\frac{d}{dx}[f(g(x))] = f'(g(x)).g'(x)$; The Chain Rule

$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1}.g'(x)$; The power rule combined with the chain rule:

$\frac{d}{dx}[\sin x] = \cos x$ $\quad$ $\frac{d}{dx}[\cos x] = -\sin x$ $\quad$ $\frac{d}{dx}[\tan x] = \sec^2 x$

$\frac{d}{dx}[\sec x] = \sec x \tan x$ $\quad$ $\frac{d}{dx}[\cot x] = -\csc^2 x$ $\quad$ $\frac{d}{dx}[\csc x] = -\csc x \cot x$

$\frac{d}{dx}[e^x] = e^x$ $\quad$ $\frac{d}{dx}[\ln x] = \frac{1}{x}; \quad x > 0$

B. Basic Integration Formulas

$\int cf(x)\,dx = c\int f(x)\,dx$ $\quad$ $\int k.dx = kx + C$

$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$ $\quad$ $\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$

$\int e^x\,dx = e^x + C$ $\quad$ $\int \frac{1}{x}dx = \ln|x| + C$

Integration by-parts: $\int u\,dv = uv - \int v\,du$

Volume (disk) $= \pi\int_a^b (f(x))^2\,dx$ $\quad$ **Area** $= \int_a^b (f(x) - g(x))dx$

Volume (washer) $= \pi\int_a^b \left[(f(x))^2 - (g(x))^2\right]dx$